High-order systems and dominant poles

Some systems with many poles can behave like systems with only one or two poles, provided those "dominant poles" account for most of the response.

Example : $$G(s) = \frac{891{,}000}{(s+1)(s+10)(s+100)}$$ PFE

$$= \frac{1000}{s+1} - \frac{1100}{s+10} + \frac{100}{s+100}$$

canonical form.

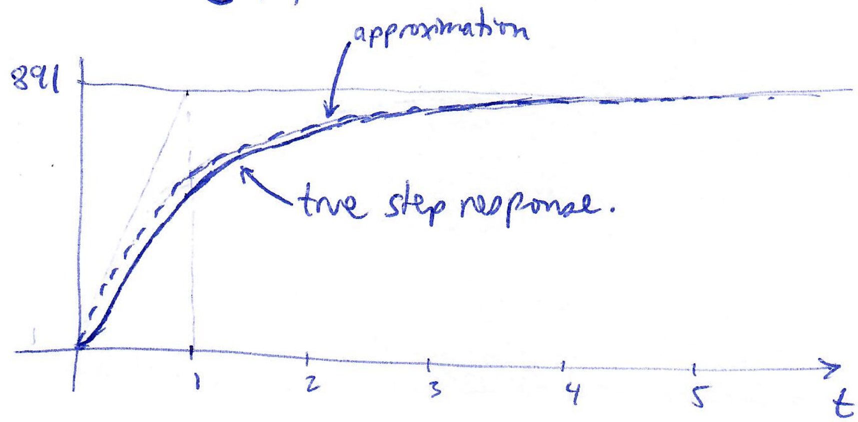
$$= \frac{1000}{s+1} + \frac{-110}{0.1s+1} + \frac{1}{0.01s+1}$$

If we look at the step response of $G(s)$, it is the sum of the step responses of each part. The final (steady state) value is $1000 - 110 + 1 = 891$.

The poles at $s=-10$ and $s=-100$ are much faster than the pole at $s=-1$. AND, they contribute a small fraction of the final steady state.

We can approximate the response of $G(s)$ by ignoring the faster poles but using the overall correct steady state value.

i.e., $$G(s) \approx \frac{891}{s+1}.$$



In general, (this is a rule of thumb) you can ignore higher order poles if their <u>real part</u> is separated by a factor of at least 5. For example:

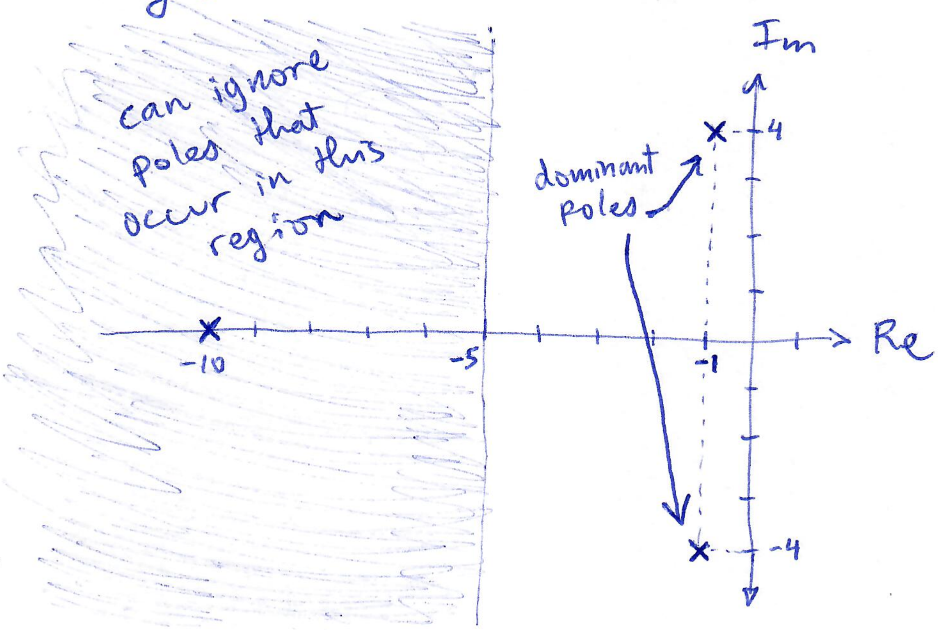
$$G(s) = \frac{1}{(s+10)(s^2+2s+17)}$$

$$= \underbrace{\left(\frac{1}{170}\right)}_{\text{gain}} \underbrace{\left(\frac{1}{\frac{1}{10}s + 1}\right)}_{\substack{\text{canonical 1st} \\ \text{order system,} \\ \text{pole @ } s=-10}} \underbrace{\left(\frac{17}{s^2+2s+17}\right)}_{\substack{\text{canonical 2nd order} \\ \text{system, poles @} \\ s=-1\pm 4j}}$$

Since the 2nd order part has poles that are 10x faster than the other pole (real part "−1" is 10x smaller than "−10"), the poles at $s=-1\pm 4j$ are <u>dominant</u>.

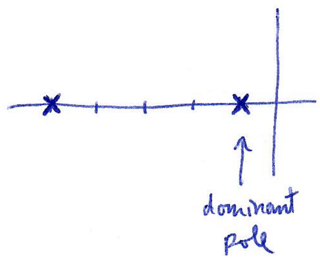
We can write $G(s) \approx \underbrace{\left(\frac{1}{170}\right)}_{\text{same gain}} \cdot \left(\frac{17}{s^2+2s+17}\right) = \boxed{\frac{1/10}{s^2+2s+17}}$

Visually, this looks like:



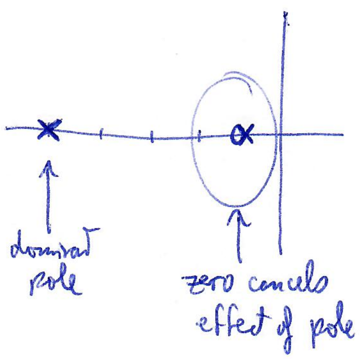
Can ignore poles for which $Re(s) \leq -5$ since they are dominated by the dominant poles with real part −1.

<u>Caution</u>: zeros can affect whether a pole is dominant or not. In general, zeros <u>diminish</u> the effect of nearby poles. The closer they are, the stronger the effect. For example:



$$\frac{1}{(s+1)(s+5)} = \underbrace{\frac{0.25}{s+1}}_{\text{dominant effect}} - \frac{0.05}{0.2s+1} \approx \boxed{\frac{0.2}{s+1}}$$

But if we add a zero close to the pole at $s=-1$...



$$\frac{s+1.1}{(s+1)(s+5)} = \frac{0.025}{s+1} + \underbrace{\frac{0.195}{0.2s+1}}_{\text{dominant effect}} \approx \boxed{\frac{0.22}{0.2s+1}}$$

Of course, if a zero were right on top of a pole, then it would cancel it exactly. For example:

$$\frac{s+1}{(s+1)(s+5)} = \frac{1}{s+5} = \frac{0.2}{0.2s+1}$$

See demo: "dominant poles"